Workshop on "Pluripotential Theory and Holomorphic Foliations"

October 25th-27th, Valenciennes

The talks will take place in Amphitheater 70E in the building Abel de Pujol 2 (Tram station : Université or Moriamez Recherche on line A, direction : Famars/Université). Lunch Break will take place at the restaurant of the university in front of the building and the coffee breaks will take place in a meeting room close by the amphitheater.

Wednesday, October 25th

09h–09h20 **Welcome**.

9h20–10h10 **Alexandre Sukhov (Université de Lille)**.

On the Kobayashi metric on Riemannian manifolds.

We introduce and study main properties of the Kobayashi metric on a Riemannian manifold. The presence of a complex or almost complex structure is not required. This is a joint work with H.Gaussier (Grenoble)

10h20-10h40 **Coffee Break**

10h40–11h30 **Thomas Pawlaschyk (Bergische Universität Wuppertal)**.

On foliations of continuous q-pseudoconcave graphs

In my talk I will discuss a joint result from 2015 with Nikolay Shcherbina on the analytic structure of certain n-pseudoconcave surfaces : the graph of a continuous map $f : G \to \mathbb{R}^k_v \times \mathbb{C}^p_\zeta$, defined on an open set G in $\mathbb{C}^n_z \times \mathbb{R}^k_u$, is locally foliated by complex n-dimensional submanifolds if and only if its complement is n-pseudoconvex in $(G+i\mathbb{R}^k_v)\times\mathbb{C}^p_\zeta$. Notice that the graph is a subset in $\mathbb{C}^n_z\times\mathbb{C}^k_{u+iv}\times\mathbb{C}^p_\zeta$. The n-pseudoconvexity is meant in the sense of Rothstein (1955). The result generalizes statements on the holomorphic structure of continuous pseudoconcave graphs of functions by Hartogs (1909, case $k = 0$, $p = 1$), Shcherbina (1993, case $n = k = 1$, $p = 0$) and Chirka (2001, case $n > 1$, $k = 1$, $p = 0$). I will also compare it to Takeo Ohsawa's similar statement from 2020 (case $k = 0$). In contrast to the more elementary proof of our result, Ohsawa's proof uses L^2-methods.

11h40–12h10 **François Bacher (Université de Lille)**.

Heat diffusions and ergodic theory for hyperbolic holomorphic foliations

In order to study the global dynamics of a holomorphic foliation by curves, we want to build an adapted ergodic theory. In particular, we need invariant measures. In most of interesting case, there doesn't exist any, and we have to consider the weaker notion of harmonic measure. Therefore, we want to study heat diffusions as natural processes that transform initial distributions into dynamically invariant distributions. In the case of hyperbolic leaves, we have two approaches to define such heat diffusions. The first one is to consider a heat kernel on any hyperbolic Riemann surface and in particular on any immersed leaf ; the second one is to solve this equation abstractly by Hille-Yosida's theorem. Both of these semi-groups of operators defined that way have given a series of ergodic theorems. We want to know when we can unify these theories by showing that these diffusions coincide. Under assumptions on the singularities of the foliation, we will discuss when we can ensure such a coincidence.

12h20–14h00 **Lunch Break**

14h00–14h50 **Sahil Gehlawat (Université de Lille)**.

The leafwise Poincare metric of a singular Riemann surface foliation.

We consider singular holomorphic foliations $\mathcal{F}$ of dimension 1 on a complex manifold M with all leaves being hyperbolic Riemann surfaces. Consider the Poincare metric λ_L on each leaf. It is known to vary smoothly along the leaves and conjectured to vary continuously along the transverse directions. There has been a lot of work on achieving this regularity of this leafwise Poincare metric for certain special cases (most of this is for foliations with discrete singular sets). The study of the regularity of this metric is equivalent to studying the Verjovsky's modulus of uniformization map η, which is a positive map defined away from the singular set E of the foliation $\mathcal{F}$. In this talk, we study this map η for foliations without any restriction on the dimension of the singular set. We will give some sufficient conditions for the continuity of the map η on the non-singular set, and also its continuous extension on the singular points. By restricting the foliation $\mathcal{F}$ to a domain $U \subset M$, we have the corresponding modulus of uniformization map η_U. We will also talk about the variation of η_U when the domain U varies in the Hausdorff sense. This is a joint work with Kaushal Verma.

15h00-15h20 **Coffee Break**

15h20–15h50 **Chin-Chia Chang (Universität zu Köln)**.

Deformation quantization and the third coefficient of Berezin-Toeplitz star product

Deformation quantization is a procedure that transforms the algebra of classical observables into the algebra of quantum operators. The usual commutative product turns into a non-commutative product known as the "star product." My focus will be on the star product derived from Berezin-Toeplitz quantization. In this talk, I will introduce Berezin-Toeplitz quantization on the compact Kähler manifolds then present my result on the computation of the coefficients of the star product and some further applications.

16h–16h30 **Thomas Kurbach (Bergische Universität Wuppertal)**.

Relative Riemann-Hilbert and Newlander-Nirenberg Theorems on some singular spaces.

The classical Relative Riemann-Hilbert Theorem on submersions states that flat relative connections are entirely determined by their kernel. In this talk, some progress is presented, where the submersion is replaced by a locally trivial morphism with singular fibers. The methods developed in the singular setting rely on considerations of reducedness, torsion-freeness and maximality of the underlying space. The existence of weak solutions is also discussed and further it is discussed under which assumptions the weak solutions are actually strong solutions. As an application it is presented, that real analytic generalized d-bar-operators can be viewed as relative connections on the complexification and hence real analytic Newlander-Nirenberg Theorems on singular spaces can be proven by solving the associated Relative Riemann-Hilbert Theorems.

Thursday, October 26th

9h–9h50 **Bingxiao Liu (Universität zu Köln)**.

Random holomorphic sections on noncompact complex manifolds

For a compact Kähler manifold, by considering the high tensor powers of a prequantum line bundle, Shiffman and Zelditch (1999) proved the equidistribution of the zeros of random holomorphic sections in the semiclassical limit. Since then, several generalizations and extensions of this result have been made in different geometric or probabilistic settings. Notably, the large deviation estimate and hole probability associated with the random zeros were obtained for compact Hermitian manifolds. In this talk, I will present a generalization of these results to the case of noncompact complex manifolds. Specifically, we will discuss the general construction of the Gaussian random holomorphic sections for Hermitian holomorphic line bundles on a noncompact Hermitian manifold. Our primary focus is on the scenarios where the space of square integrable holomorphic sections is infinite-dimensional. Then we investigate the behaviours of their zeros in the semiclassical limit, covering topics such as equidistribution, large deviation estimate and hole probability. This talk is based on the joint work with Alexander Drewitz and George Marinescu.

10h00–10h30 **Coffee Break**.

10h30–11h20 **Nikhil Savale (Universität zu Köln)**.
TBA

11h30–12h00 **Paul Hinkelmann (Bergische Universität Wuppertal)**.

The Ohsawa-Takegoshi extension theorem

Given a L^2-holomorpic function f on some analytic subset of a pseudoconvex domain, one can ask for a holomorphic extension F of f to the whole space together with good L^2-estimates for F. This type of extension theorems are considered as Ohsawa-Takegoshi type extension theorems and first results were mentioned by Ohsawa in 1987. In this talk we are going to discuss a version which was proved by B. Berndtsson and L. Lempert in 2014, using plurisubharmonic variation of the given domain.

12h10–14h **Lunch Break**.

14h–14h30 **Bergur Snorasson (Háskóli Íslands)**.

A weighted Bernstein-Walsh-Siciak theorem with polynomials associated to convex sets

We generalize the Bernstein-Walsh-Siciak theorem on polynomial approximation to the case where the standard polynomial ring is replaced by a subring consisting of all polynomials with exponents restricted to sets mS, where S is a compact convex subset of $\mathbb{R}^n_+$ with $0 \in S$ and $m = 0, 1, 2, 3, ...$, and uniform estimates of error in the approximation are replaced by weighted uniform estimates with respect to an admissible weight function.

15h–15h30 **Coffee Break**.

15h–15h30 **Álfheidur Edda Sigurdardóttir (Háskóli Íslands)**.

Pluripotential theory with respect to convex sets - A Siciak-Zakharyuta theorem

A polynomial is of degree m if its exponents are contained in the m-th dilate of the unit simplex. If the unit simplex is replaced by any compact convex set S, we obtain a new grading of the polynomial ring. We work with a Siciak function with respect to the S-grading. That function generally has different growth than the usual logrithmic growth, so we work with a Siciak-Zakharyuta/pluricomplex Green function with matching growth. We show that the sets S that yield a generalization of the Siciak-Zakharyuta theorem are precisely the sets whose rationals form a dense subset.

15h40–16h10 **Shuang Su (Universität zu Köln)**.

Obstructions of currents to be full mass intersection

Lelong number is an important notion to measure the singularities of currents. The study of currents to be full mass intersection on a compact Kähler manifold is closely related to the Lelong numbers of the given currents. In this talk, I will first review some basic properties of Lelong number and non-pluripolar product. Then, I will talk about some results about full mass intersection.Finally, I will talk about some recent works with Duc Viet Vu. We consider an irreducible analytic subset of a compact Kähler manifold, and try to study the obstruction of currents to be full mass intersection on the analytic subset.

16h20–17h00 **Problem Session**. *Held by Judith Brinkschulte*

Group picture

19h00 **Dinner** *Chez mon vieux, 3 Rue Derrière la Tour, 59300 Valenciennes*

Friday, October 27th

9h00-9h50 **Josias Reppekus (Bergische Universität Wuppertal)**

One-resonance and doubly connected basins of attraction in dynamics of iterated holomorphic maps

A holomorphic map is locally given by its power series expansion. Thus, the local dynamics of the an iterated holomorphic map near a fixed point can often be determined from a finite number of terms of the expansion at the fixed point. Local invariant sets with locally stable dynamics can then be extended via backward images to global objects with the same long-term dynamics. For polynomials on $\mathbb{C}$, stable dynamics near fixed points are well understood and all stable dynamics arise from fixed points. In $\mathbb{C}^2$ both local and global stable dynamics still pose many open questions.
New types of dynamics at neutral fixed points in $\mathbb{C}^2$ arise from non-trivial multiplicative relations of eigenvalues in the linear part, called resonances. In this talk I will present a construction of examples in $\mathbb{C}^2$ with a product of eigenvalues equal to 1. In this so-called one-resonant case, we obtain a non-linear projection to one variable, allowing us to construct a doubly connected attracting open set. To show the "hole" cannot be filled to obtain a larger simply connected attracting set, we impose a small divisor condition on the eigenvalues and show that our attracting set is the whole basin of attraction of the fixed point.

10h00-10h30 **Coffee Break**

10h30-11h20 **Ood Shabtaï (Université de Lille)**

Off-diagonal estimates of partial Bergman kernels on S^1-symmetric Kahler manifolds

We show that partial Bergman kernels on closed, connected S^1-symmetric Kähler manifolds admit point-wise asymptotic expansions, and compute the leading order term. The results complement the existing, on-diagonal estimates of partial Bergman kernels.

11h30-12h00 **Wei-Chuan Shen (Universität zu Köln)**

Semi-classical spectral asymptotics of Toeplitz operators on CR manifolds

Let X be a compact strictly pseudoconvex embeddable CR manifold and let T_P be the Toeplitz operator on X associated with a first order pseudodifferential operator P. We consider the operator $\chi_k(T_P)$ defined by functional calculus of T_P, where χ is a smooth function with compact support in the positive real line and $\chi_k(\lambda) := \chi(k^{-1}\lambda)$. We will show that the kernel of $\chi_k(T_P)$ is a semi-classical Fourier integral modulo a k - negligible smooth kernel. Time permitting, I will give some applications that provide CR analogues of asymptotics for powers of line bundles in complex geometry.

12h10-14h00 **Lunch Break**

14h30-16h00 **Guided Tour (in english) :** ***Valenciennes in the XVIIIth century***
The tour starts in front of the library of Valenciennes. The best way to go from the university is by tram, stop at "hotel de Ville" and walk almost straight to the library called after Simone Veil.